Mathematics: analysis and approaches SL

Timezone 1

To protect the integrity of the assessments, increasing use is being made of examination variants. By using variants of the same examination, students in one part of the world will not always be responding to the same examination content as students in other parts of the world. A rigorous process is applied to ensure that the content across all variants is comparable in terms of difficulty and syllabus coverage. In addition, measures are taken during the standardisation and grade awarding processes to ensure that the final grade awarded to students is comparable.

Contents

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Grade boundaries

Standard level overall

Grade:	1	2	3	4	5	6	7
Mark range:	0-9	10-21	22-32	33-46	47-62	63-75	76-100

Standard level internal assessment

Grade:	1	2	3	4	5	6	7
Mark range:	0-2	3-5	6-8	9-11	12-14	15-17	18-20

Standard level paper one

Grade:	1	2	3	4	5	6	7
Mark range:	0-8	9-17	18-25	26-35	36-48	49-58	59-80

Standard level paper two

Grade:	1	2	3	4	5	6	7
Mark range:	0-7	8-16	17-24	25-35	36-48	49-58	59-80

Standard level internal assessment

The range and suitability of the work submitted

The majority of students successfully implemented practical applications of mathematics, showcasing their ability to connect theoretical concepts with real-world scenarios, models and data. The mathematical techniques employed by the students were mostly consistent with the academic level of the course, ensuring that the explorations remained within an appropriate difficulty range while still challenging the students to apply their knowledge effectively.

Many explorations relate to statistics, primarily modelling and correlation with the two largest categories of topics being sports and economics/business/social science. Students do seem to have a genuine interest in these topics treated in this way and it is clearly accessible to them as a method. It is encouraging, though, that more interesting and original explorations with a varied range of aims and approaches still get completed by students and these are often refreshing to read! It is clear that the vast majority of schools clearly encouraged students to choose their own topics but there are definitely still schools that provide students with a template for completion.

There were certainly and perhaps unsurprisingly less SEIR/COVID modelling related explorations. There seems to be a slight increase in 'scientific' explorations that follow the structure of a science practical write up. Explorations of pure mathematics remain rare. Students should avoid writing research-style explorations, as they regrettably often lead to a focus on presenting other people's work, rather than "using the mathematics" themselves.

There were relatively few occurrences of incomplete or very low scoring work.

Student performance against each criterion

Criterion A

Most students are able to access A3/A4. Coherent, well-organized explorations typically include an introduction that sets the context, a clear description of the aim, and a concluding section that summarized the findings but concision remains challenging for many. However, some students do not seem to have a good idea about who their intended audience is. They should be reminded that their explorations should be understandable to a classmate in their course, who is not expected to know mathematics outside the course without clear explanation. If an AASL student is unlikely to understand with the explanation provided, then the work is not likely to be judged as entirely coherent. Students often do well with defining non-mathematical terminology specific to the topics they investigate. Students are reminded that they are not expected to explain the mathematics for prior learning topics e.g. how to calculate a mean, range etc.

Criterion B

The students seem to be improving in their use of an equation editor to produce appropriately formatted mathematics, although a few students seem to go out of their way to avoid it's use. The text in graphs and images is often significantly smaller than the body text and on the IB marking software can be hard to read. Attention to detail (defining variables, labelling graphs, using approx. equals signs, consistent and appropriate degree of accuracy etc) easily allow students access to higher marks but still a surprising number are making avoidable mistakes, e.g. for multiplication, computer notation, not bothering with subscripts etc. A careful read of a few years of subject reports will help students see what to avoid!

Criterion C

Teachers are mostly correctly interpreting this criterion to be engagement with the mathematics rather than just with the real-world activity that led to the investigation or worse still rewarding hard work and effort, but these misunderstandings of the criteria are still sometimes a problem for individual schools. However many students are actually considering alternative approaches to confirm their first methods, engaging well with mathematical methodology, thinking independently, making and testing predictions, presenting their mathematical ideas in unique and interesting ways and exploring topics from multiple perspectives.

Criterion D

Few students reach the top mark here. Many reflections are quite generic and lack depth rather than thinking critically about their own exploration, data, use of mathematics etc. This substantial and critical reflection that drives the next steps of the exploration is rare. Reflecting on, and addressing, how the results of their analysis affect the aims, or why a certain analysis was unsuccessful, or leads to an inaccurate prediction/assumption and how this impacts the choice of the next steps, or reflecting and choosing another way to analyse the problem, remain areas in need of attention/practice. Students often fail to notice the inconsistencies in their own calculations.

Criterion E

Teachers are reminded to check the student work for mathematical errors. The student understanding of the mathematics must be “demonstrated” in the exploration itself. Repeating a calculation from a source without showing an understanding of its derivation, or why it describes/defines the mathematics it represents does not give evidence for the student’s own understanding. Some ways of providing that evidence are explaining the concepts in their own words or creating their own examples. Equally, some students are still using technology to produce a model without explaining their choice of function. Insufficient consideration is given to a behaviour before and after the domain of the data set from which it was generated. An emphasis on the Modelling process of first plotting the data and then an analytical consideration of the behaviours/key points of each function before using analytical, or regression, methods to fit a function, is desirable. Presenting numerous models, without justification or reflection on the above considerations, is unlikely to earn the highest attainment levels.

Doing a page full of calculations for something like the correlation coefficient or parameters of a linear model will not, by itself, improve the score in this criterion. Students often prioritize calculations over understanding, lacking depth in mathematical comprehension. Clear explanations of methods' significance are essential to elevate the exploration quality.

Recommendations and guidance for the teaching of future students

Students should be made aware of the assessment criteria before they begin drafting their work. A clear understanding of the criteria enables students to align their explorations with the expected standards, ensuring that they address all necessary components comprehensively and coherently.

Schools should tell students to line-space their work with standard margins and not attempt to either shorten or lengthen their explorations by adjusting spacing and font size. The suggested length of 12 to 20 pages is not a hard and fast rule and can vary dependent on exploration type.

Page numbers are helpful if included in the document. Students should be reminded that the cover page needs a title and page count only.

Students should be reminded that all sources must be appropriately cited not only in the bibliography, but in the text where the information is used, and all in-text citations should have a corresponding bibliographic entry. Citations should pinpoint which ideas come from a source. In addition, teachers must check sources. If a moderator can readily find plagiarism, then the teacher, who has worked with the student from the beginning of the process, should be expected to find it first, before the exploration gets to this stage.

Handwriting comments or commenting directly on to electronic versions of the student work are both appropriate but note that the scanning process can produce hard to read comments and some electronic highlighting and comments can obscure the student text.

Teachers may find it helpful to share some of the 40+ examples on the IB's "Mathematics Assessed Student Work" in My IB > TSM area.

Students should be reminded to redact all personal info and teachers should be aware this is also the case on a marking sheet.

Further comments

QR codes/links that point to online material to illustrate points in the exploration are unfortunately not considered in the marking.

A few noticeable and some possible cases of the use of AI. This is definitely something teachers and moderators should and will keep an eye on.

A number of internal assessments are still being submitted without teacher commentaries and/or annotation directly on the student's work. This is frustrating to the moderator and means they are less likely to be able to confirm the teacher mark.

Standard level paper one

The areas of the programme and examination which appeared difficult for the students

Using an unfamiliar pattern/sequence, determining the number of solutions using a sketch, integration of $\sin 2x$, finding the equation of a normal, using the second derivative test to justify a stationary point, properties of logarithms.

The areas of the programme and examination in which students appeared well prepared

Basic arithmetic and geometric sequences, differentiating polynomial functions, basic probability, one-variable statistics, using derivatives to find stationary points and points of inflection, solving trigonometric equations.

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

Very well done overall. Most students were able to find the difference and the general term of the arithmetic sequence.

Question 2

Generally, well done. Some students struggled to find the correct mean equation by dividing by 6 instead of 16, or by not multiplying score and frequency. If only one equation was found, students often guessed values of p and q; full marks were not earned in this case. If there are two variables to determine, students can expect to use two equations. To determine the final mean in part (b) many students re-calculated the mean, which took more time, rather than using transformations understanding of the data being multiplied by a constant.

Question 3

Manipulating logarithms seemed to be beyond many students who struggled to execute the basic techniques. In part (a) some students used the property of logarithms correctly but failed to recognize that $\log 1 = 0$. Part (b) presented even more difficulty with few students successful in using the change of base formula. Some opted for an indices approach and were typically successful when doing so.

Question 4

Students typically started part (a) well by forming the correct area equation and mostly the correct perimeter. However, correctly eliminating θ was more challenging. In part (b) most students correctly factorised the equation given in part (a) to find r. Again, arithmetic with fractions was problematic.

Question 5

Solving the trigonometric equation with a double angle rule was well done, even by weaker students. It was encouraging to see many attempt to find the area between the curves, however, many students did not successfully integrate $\sin 2x$ correctly. Many students earned both method marks for recognizing the general approach required.

Question 6

Most students correctly found an expression for S_n. Part (b) proved to be the most difficult question in the whole paper with few students recognizing the pattern of the sums. Some students simplified the values of S_1, S_2, S_3 thus trying to work with 1, 11, 111, … but were unable to progress. This inquiry approach was significantly challenging for students.

Question 7

Many students scored well in all the parts of the question. The most challenging part of the question was using the second derivative test in part (b) where many students overlooked the instructions in the question. Using a sign diagram to mimic the definition of a result that the question already provides will not earn full marks. In this case, a specific value must be found using the second derivative to justify the result that A is a local maximum. The fraction arithmetic also proved challenging. In part (c) many students wasted time finding the y-coordinate.

Question 8

The clarity and quality of responses to sketching the rational function varied considerably. Despite clear instructions to label features of their graph some students did not. Typically, students wrote the asymptote equations correctly e.g. $x = 2$ instead of previously often seen $x \neq 2$ which was good to see. It was quite common for only one branch of the rational function to be sketched. Students were quite successful in parts (d) and (e). Recognizing that a sketch can be used in part (f) to determine the number of solutions to $f(x) = g(x)$ was rare. Most students spent time trying to solve the equation algebraically and then mistakenly assuming that every cubic equation will have 3 real solutions.

Question 9

Parts (a) and (b) were successful for most students. Part (c) was well done by those students who recognized conditional probability. Many were successful with the probability distribution table, including follow through with the expected number for incorrect values of a and b. Most students recognized $P(next\ blue) = \frac{1}{6}$ but did not read the question properly to provide this as their answer. Part (g) was mostly attempted using a numerical approach. Some students recognized that four red buttons were first selected but perhaps missed the phrase 'in total', in the stem of the question. Thus stating $n = 4$ instead of $n = 5$ as their answer.

Recommendations and guidance for the teaching of future students

Inquiry approaches should feature throughout the teaching of the course to assist students in seeking patterns and dealing with unfamiliar problems. There were a couple of opportunities in the paper for students to demonstrate this, but they seemed poorly prepared to do so. All the topics of the syllabus should be taught and consolidated using connections between them. Past examination questions provide plenty of practice in this regard and students should be exposed to questions of this type as soon as possible in their course.

Teachers should encourage students to look at the ‘flow’ of the longer problems and see how earlier parts are helpful to aid completion of later parts. Also to reiterate to their students the need to read specific directions carefully. Knowing the command terms is also essential for understanding the expectations of a question.

Good communication is helpful. Disorganized work can be difficult to decipher. Work should be presented in a logical and coherent order, finishing with the answer. Many examiners commented on the work being difficult to follow, including poor penmanship. Examiners cannot mark what they cannot read. Similarly, if multiple solutions are offered, cross out which is not to be read, otherwise the first solution will be marked. Do not cross out any work unless it is replaced by other working.

Standard level paper two

The areas of the programme and examination which appeared difficult for the students

These include:

- working with sufficient accuracy in order to obtain a correct 3 sf answer
- interpreting and comparing statistics given in context
- application of trigonometry in geometric shapes
- problem solving with a normal distribution
- interpreting a velocity-time graph, and using the GDC to calculate displacement
- limitations to using the equation of a regression line to make predictions
- understanding notation, e.g. PQ, and use of key vocabulary, e.g. extrapolation
- determining the domain of a function after a transformation
- solving exponential equations analytically
- converting between units of time
- determining the parameters of a trigonometric function
- real life applications of the ambiguous case of the sine rule

The areas of the programme and examination in which students appeared well prepared

These include:

- reading values on a box and whisker plot
- simple compound interest problems involving a decrease in value
- finding the volume of a cone
- use of the GDC to find Pearson's product-moment correlation coefficient
- finding the inverse of an exponential function.
- transformations of functions
- use of the sine and cosine rules
- area of a non-right angled triangle
- using the GDC to find key features of a graph

The strengths and weaknesses of the students in the treatment of individual questions

Question 1

Part (a) provided a relatively straightforward start to this paper. Most students achieved all three marks available, being able to correctly find the median, lower quartile and upper quartile from the given boxplot, and to calculate the interquartile range.

While a few students were able to give concise, accurate responses in part (b), in general, the reasoning behind the judgments made by most students was lacking. Many did not make a comparison between either the median or the IQR of the two countries e.g. which had the larger median ear length, or the higher

IQR. Of those that did, most used imprecise terminology to interpret the meaning of their comparisons in the context of the question. There was a tendency to make sweeping $n=10$ generalizations about each country's distribution of ear length than could be reasonably inferred from the provided information.

Question 2

Most students appeared to be familiar with this type of finance question, and were successful in part (a). Most correctly identified the multiplier of 0.85 to obtain $29750, which they then used in their equation in part (b). A very common error in part (b) was for students to fail to take into account the first year from part (a), and to use $n=10$ rather than $n=9$ in the compound interest formula.

Those that were successful in parts (a) and (b), were usually able to set up a correct initial equation or inequality in part (c). While a few concise solutions using the GDC were seen, most proceeded to solve this algebraically using logarithms, and found the first two marks relatively straightforward. If errors occurred, they were often due to premature rounding. The last mark proved more challenging, with an incorrect answer of $n=19$ commonly seen.

Few students used a Finance application on their GDC. A small number of students attempted to solve parts (b) and (c) by repeated multiplication, finding the value of the car after each complete year. This approach was usually poorly communicated and unsuccessful.

Question 3

Most students recognised the need to use trigonometry to find the value of $r+5$. However, despite the triangle having a right-angle, the majority opted to use the sine rule. Though a valid method, fewer students appeared to be successful with this approach compared to those that attempted to use a simple trigometric ratio. The most common error was to ignore the height of the person and use 20, rather than 18.2, as a side of the triangle. Although a few students extended the hypotenuse to the ground, it was rare to see any acknowledgment that by doing so, the horizontal distance had consequently increased. In part (b), most students confidently substituted their radius into the formula for the volume of a right cone.

Question 4

A few elegant and concise solutions were seen to this question by students who had a strong understanding of the normal distribution, or who were able to effectively use their GDC. Those that recognised that the standardized normal distribution could be used in both parts were able to quickly and accurately find $\mathrm{P}(Z>1.5)$, and solve $\mathrm{P}(Z>k)=0.1$. However, many students found this question challenging and either struggled to understand how to proceed or did not attempt either part.

Very few attempted to sketch a normal distribution curve, annotated with the given information, to help them visualize the questions. Those that did were generally more successful and were able to correctly write $\mathrm{P}(X>13)$ and $\mathrm{P}(X>10+2k)=0.1$. Without this clarity, it was not unusual to see $\mathrm{P}(X>1.5)$ or $\mathrm{P}(X>11.5)$ being considered in part (a). The answer of 0.067 reported to two significant figures was often seen. Most students who attempted part (b), recognised how to find a critical value given a probability and obtained 12.6, 7.44 or 1.28, but were not sure how to then find k.

Calculator notation was often seen, and effectively communicated the approach being taken. However, a common error was to attempt to use a normal probability density function, rather than the cumulative density function.

Question 5

There was a mixed level of understanding and proficiency with the GDC demonstrated in this question. Some students had a very clear grasp of how to interpret key features of a velocity-time graph and when this was combined with good graphic calculator skills, the solutions were efficiently found. Some students appeared to be 'tracing' the function on their GDCs rather than calculating the precise zeros, and the lack of accuracy meant they were not awarded some of the marks in parts (a) and/or (b). In part (c) many students recognised the need to integrate v but then attempted, often unsuccessfully, to do this analytically when the answer could have been obtained much more efficiently using their GDCs. Occasionally, finding the displacement was confused with calculating the distance travelled.

Common incorrect responses in part (a) included, finding the value of t at the local maximum of the given graph as being the point at which the change in direction occurred, and in part (b), identifying $v'(t) > 0$ as defining the required interval. Others found the values of t when $v(t) = 0$, but were unable to form a range of values.

Question 6

Most students were able to accurately determine Pearson's product-moment correlation coefficient in part (a) and, when they recognised in part (c)(ii) the need to use the regression line of x on y, to obtain and use the correct regression equation. Accuracy was again an issue in these parts, with 0.90 or 0.9 often seen in part (a) and premature rounding seen in part (c).

Students should be well-versed in what was required in parts (b) and (c)(i). However, it was surprising that these parts were so poorly answered, with many appearing to lack the required statistical language.

Question 7

In part (a), the majority of students confidently interchanged x and y, and were usually successful in obtaining the inverse function. Some had difficulty presenting clear work, or did not demonstrate the necessary steps that led to the given answer. Part (b) was answered well by those that understood the notation, but too few realised that the length of a line segment was required. Most found only points P and Q. Some went on to calculate an integral using the x-coordinates as limits.

Part (c)(i) was mostly well answered, although many found a horizontal translation rather than a vertical translation or, to a lesser extent, reflected in the y-axis rather than the x-axis. Inevitably, this caused follow through issues in part (d). Very few were able to find the domain in part (c)(ii), with many either not attempting it, or stating incorrectly $x \in \mathbb{R}$. The final part of this question was challenging for most, even when they had a correct answer in part (c)(i), and few gained any marks. Most struggled with the algebra required to rearrange their equation. Those that had like terms which could be combined often were unable to add their two terms, or to multiply through their equation by $\frac{2}{3}$. It was common to see students incorrectly applying logarithms to individual terms.

Question 8

Success in this question depended on the ability to interpret the question in relation to the graph of the function, as well as being able to determine key features of the graph to an appropriate level of accuracy. Students who did well in this question had often drawn sketches of the graph with key features labelled at the start, which they had then annotated as they worked through parts (a) and (b). That said, a common error in these first two parts was the premature rounding of values obtained from the calculator. Many spent too much time attempting to solve analytically $H(t)=0.5$ or $H(t)=3.76$. In part (a) many struggled to convert the times given into hours. Those that converted their times into minutes tended to be more accurate.

In part (c) the need to differentiate the function was often recognised, and many were able to obtain -0.651. However, rather than using their GDCs to find the derivative of the function at $t=13$, many students again attempted an analytical approach, which was not always successful. Part (d) was particularly well done by a small proportion of students, with some showing a real depth of understanding in calculating the values of b and c. However, the majority made little progress beyond finding the value of the parameters a and d. Many attempted to find the value of b, but it was common to see $9:02-2:41=6.61$. Few understood how to find the value of c. While much of the work in this part was poorly presented, students generally indicated very clearly their value for each of the parameters, which helped examiners to award follow through marks in part (e).

In the final part, many students equated $H(t)$ to their $h(t)$, but it was then rare to see a sketch of their graphs with a point of intersection indicated. This would have resulted in many more being awarded the method mark, rather than no marks when they did not accurately find the value of t using their GDC.

Question 9

The first part of this question was found to be accessible to most students. The majority chose to approach the problem by using the sine rule to find $\text{A}\hat{\text{B}}\text{O}=69.9^\circ$, which they then used to find $\text{O}\hat{\text{A}}\text{B}=82.1^\circ$. However, few identified the ambiguous case and found only one value for $\text{A}\hat{\text{B}}\text{O}$ and consequently only one value for $\text{O}\hat{\text{A}}\text{B}$. Where a second value for $\text{A}\hat{\text{B}}\text{O}$ was determined, it was frequently obtained by incorrectly positioning the fence from B to the wall, rather than from A to the hedge as given in the question. However, part (a)(ii) was well understood and students were generally able to use the triangle area formula with their angles to find their areas.

In part (b) many students were either able to find one equation, or to gain some marks from choosing to use the cosine rule and area formula. Appropriate values were not always fully substituted to gain further marks. Few students successfully completed this 'show that' step. Part (c) was one of the more challenging questions on the paper and only a minority of students were able to form an equation in one variable that would lead to the correct answers.

Recommendations and guidance for the teaching of future students

There were a number of questions in this paper where students did not communicate their understanding effectively using correct mathematical terminology. In the statistic questions specifically, it was not uncommon to see, in question 1, "wide range" being used instead of "larger spread", and in question 2, "outlier" when referring to any value outside of a data set, instead of "extrapolation".

Students are encouraged to take time to interpret the information given, and as appropriate, provide annotated diagrams or restate contextual questions in symbols. For example, students who took the time to draw and annotate a normal distribution curve in question 4, or to label the given sketch in question 5, consequently appeared to find those questions easier to access.

Ensure that students have a good grasp of how to perform a variety of operations on their graphics calculators. This should include applications they are not able to do easily analytically e.g. solving complex and unfamiliar equations, graphing composite functions, graphing derivative functions, evaluating derivatives and integrals.